

Problema 1, Clasa a IX-a

a) Total: 1,5p

$$t = \frac{d}{v} \quad 1 \times 0,5p$$

$$t_1 = \frac{AC}{v_1} + \frac{CB}{v_2} = 220 \text{ s} \quad 2 \times 0,5p$$

b) Total: 1,5p

$$AB = \sqrt{AC^2 + BC^2} \quad 1 \times 0,5p$$

$$t_2 = \frac{AB}{v_1} = 40\sqrt{109} = 417,6 \text{ s} \quad 2 \times 0,5p$$

c) Total: 1,5p

$$AD = \sqrt{AC^2 + DC^2} \quad 1 \times 0,5p$$

$$t_2 = \frac{AD}{v_1} + \frac{BD}{v_2} = 260 \text{ s} \quad 2 \times 0,5p$$

d) Total: 3,25p

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{1}{4} \quad 2 \times 0,5p$$

$$\sin r = 1 \quad 1 \times 0,25p$$

$$\sin^2 i + \cos^2 i = 1 \quad 1 \times 0,25p$$

$$\tan i = \frac{\sin i}{\cos i} = \frac{1}{\sqrt{15}} \quad 1 \times 0,25p$$

$$CE = AC \tan i = 2\sqrt{15} \text{ m} = 7,75 \text{ m} \quad 2 \times 0,25p$$

$$AE = \frac{AC}{\cos i} \quad 1 \times 0,25p$$

$$BE = BC - CE \quad 1 \times 0,25p$$

$$t_{\min} = \frac{AC}{v_1 \cos i} + \frac{BC - AC \tan i}{v_2} = 216,2 \text{ s} \quad 2 \times 0,25p$$

e) Total: 2,25p

Față de apă:

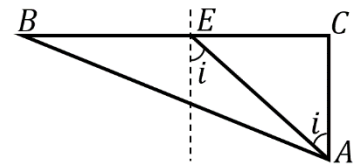
$$v_{r2} = v_2 + v_a \quad 1 \times 0,5p$$

$$\sin i = \frac{v_1}{v_2 + v_a} \quad 1 \times 0,5p$$

$$\cos i = \frac{\sqrt{(v_2 + v_a)^2 - v_1^2}}{v_2 + v_a} \quad 1 \times 0,25p$$

$$CE = \frac{AC}{v_1 \cos i} (v_a - v_1 \sin i) \quad 3 \times 0,25p$$

$$CE = 55,8 \text{ m} \quad 1 \times 0,25p$$



Problema 2, Clasa a IX-a

a) Total: 0,5 p

$$R = \rho \frac{l}{S} \quad 1 \times 0,25 \text{ p}$$

$$R = 5,0 \, \Omega \quad 1 \times 0,25 \text{ p}$$

b) Total: 1,0 p

$$R_{AB} = \frac{R_1 R_2}{R_1 + R_2} \quad 2 \times 0,25 \text{ p}$$

$$I_{AB} = \frac{U_{AB}}{R_{AB}} \quad 1 \times 0,25 \text{ p}$$

$$I_{AB} = 4,8 \text{ A} \quad 1 \times 0,25 \text{ p}$$

c) Total: 2,25 p

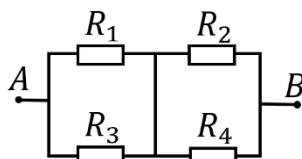
$$R_1 = R_3 = \frac{\rho x}{S} \quad 2 \times 0,25 \text{ p}$$

$$R_2 = R_4 = \frac{\rho(l-x)}{S} \quad 2 \times 0,25 \text{ p}$$

$$R_{13} = \frac{R_1}{2}, \quad R_{24} = \frac{R_2}{2} \quad 2 \times 0,25 \text{ p}$$

$$R_{AB} = R_{13} + R_{24} = \frac{R}{2} \quad 2 \times 0,25 \text{ p}$$

$$I_{AB} = 4,8 \text{ A} \quad 1 \times 0,25 \text{ p}$$



Pentru scrierea rezultatului final corect, fără demonstrare, se acordă 0,5 p

d) Total: 2,5 p

$$R_1 = R_4 = \frac{\rho x_1}{S} = \frac{R}{4} \quad 3 \times 0,25 \text{ p}$$

$$R_2 = R_3 = \frac{\rho x_2}{S} = \frac{3R}{4} \quad 3 \times 0,25 \text{ p}$$

$$R_{13} = R_{24} = \frac{3R}{16} \quad 2 \times 0,25 \text{ p}$$

$$R_{AB} = \frac{3R}{8} \quad 1 \times 0,25 \text{ p}$$

$$I_{AB} = 6,4 \text{ A} \quad 1 \times 0,25 \text{ p}$$

e) Total: 1,75 p

$$U_{13} = U_{24} = \frac{U_{AB}}{2} \quad 2 \times 0,25 \text{ p}$$

$$I_1 = \frac{2U_{AB}}{R} \quad 1 \times 0,25 \text{ p}$$

$$I_2 = \frac{2U_{AB}}{3R} \quad 1 \times 0,25 \text{ p}$$

$$I_c = I_1 - I_2 = \frac{4U_{AB}}{3R} \quad 2 \times 0,25 \text{ p}$$

$$I_c = 3,2 \text{ A} \quad 1 \times 0,25 \text{ p}$$

f) Total: 2,0 p

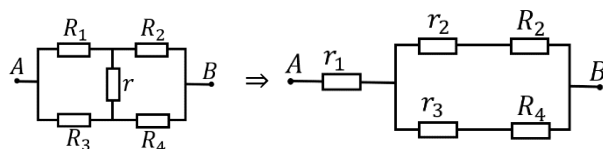
$$r_1 = \frac{R_1 R_3}{R_1 + R_3 + r} = \frac{3R}{20} \quad 1 \times 0,25 \text{ p}$$

$$r_2 = \frac{R_1 r}{R_1 + R_3 + r} = \frac{R}{20} \quad 1 \times 0,25 \text{ p}$$

$$r_3 = \frac{R_3 r}{R_1 + R_3 + r} = \frac{3R}{20} \quad 1 \times 0,25 \text{ p}$$

$$R_{AB} = r_1 + \frac{(r_2 + R_2)(r_3 + R_4)}{r_2 + r_3 + R_2 + R_4} = \frac{5R}{12} \quad 4 \times 0,25 \text{ p}$$

$$I_{AB} = \frac{144}{25} \text{ A} = 5.8 \text{ A} \quad 1 \times 0,25 \text{ p}$$



Problema 3, Clasa a IX-a

a) Total: 2,0 p

Desen corect (raza incidentă, refractată, emergentă, normale în punctele de refracție) -

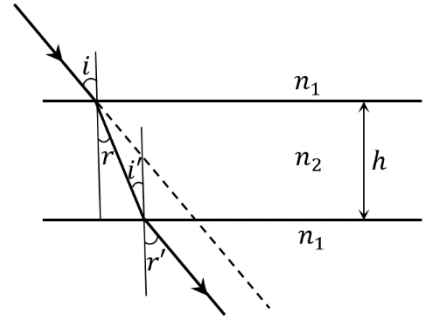
$$\frac{\sin(i)}{\sin(r)} = \frac{n_2}{n_1} \quad 1x0,25p$$

$$i' = r \quad 1x0,25p$$

$$\frac{\sin(i')}{\sin(r')} = \frac{n_1}{n_2} \quad 1x0,25p$$

$$r' = i \Rightarrow \quad 1x0,25p$$

Raza emergentă și incidentă sunt paralele.



b) Total: 2,5p

Desen corect (distanța dintre raze) -

$$n_2 = n, n_1 = 1 \quad 1x0,25p$$

$$\sin(r) = \frac{\sin(i)}{n} \quad 2x0,25p$$

$$\cos(r) = \sqrt{1 - \frac{\sin^2 i}{n^2}} \quad 1x0,25p$$

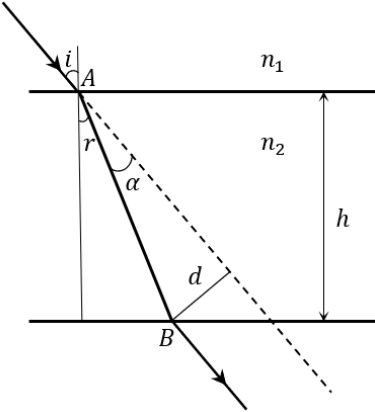
$$\alpha = i - r \quad 1x0,25p$$

$$\frac{d}{AB} = \sin(\alpha) \quad 1x0,25p$$

$$AB = \frac{h}{\cos r} \quad 1x0,25p$$

$$d = h \sin(i) \left(1 - \frac{\cos i}{\sqrt{n^2 - \sin^2 i}}\right) \quad 1x0,25p$$

$$d = 0.58 \text{ cm} \quad 1x0,25p$$



c) Total: 3,25p

Desen corect (mersul a două raze de la sursa punctiformă și prezentarea imaginii la intersecția prelungirilor razelor) -

$$SI = h + SA - IC \quad 3x0,25p$$

$$tgi = \frac{AB}{SA} \Rightarrow SA = \frac{AB}{tgi} \quad 2x0,25p$$

$$tgi = \frac{DC}{IC} \Rightarrow IC = \frac{DC}{tgi} \quad 1x0,25p$$

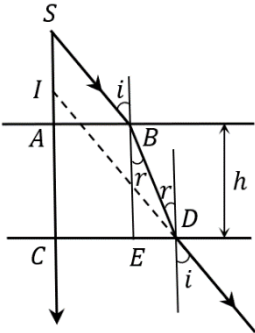
$$DC - AB = DE \quad 1x0,25p$$

$$DE = h \tan(r) \quad 1x0,25p$$

$$SI = h - \frac{DC - AB}{tgi} = h - \frac{DE}{tgi} \quad 1x0,25p$$

$$SI = h \left(1 - \frac{tgr}{tgi}\right) \approx h \left(1 - \frac{\sin r}{\sin i}\right) \quad 2x0,25p$$

$$SI = h \left(1 - \frac{1}{n}\right) = 1,0 \text{ cm} \quad 2x0,25p$$



d) Total: 0,75p

$$H = h - h \left(1 - \frac{1}{n}\right) = \frac{h}{n} = 2,0 \text{ cm} \quad 3x0,25p$$

e) Total: 1,5p

$$y_1 = h_1 \left(1 - \frac{n_2}{n_1}\right) \quad 1x0,25p$$

$$\dots$$

$$y_i = h_i \left(1 - \frac{n_{i+1}}{n_i}\right), \text{ pentru } i = 1 \dots m \quad 1x0,25p$$

$$n_{m+1} = 1 \quad 1x0,25p$$

$$y = y_1 + y_2 + \dots + y_m \quad 1x0,25p$$

$$H = h_1 + h_2 + \dots + h_m \quad 1x0,25p$$

$$d = H - y = \frac{h_1 n_2}{n_1} + \frac{h_2 n_3}{n_2} + \dots + \frac{h_m}{n_m} \quad 1x0,25p$$